1856MNRAS..16..139M

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MANUEL J. JOHNSON, Esq., President, in the Chair.

Rev. W. Jackson, M.A., St. John's, Workington; and
Geo. Roberts Smalley, Esq., Head Mathematical Master in King's College School,
were balloted for and duly elected Fellows of the Society.

On the Values of the Diameters of the Planets having measurable Disks, as determined with a Double-Image Micrometer attached to the East Equatoreal of the Royal Observatory, Greenwich. By the Rev. R. Main, M.A.

"The observations, of which the results are here discussed, extend over a long space of time,—namely, from the time of the invention of the double-image micrometer by Mr. Airy in 1840 till the year 1851. At the commencement of the use of this instrument, it was considered that one of the most important applications of it would be for the determination of the diameters of planets, its range extending considerably beyond that required for the measurement of the system of *Saturn*, and its accuracy within that range being far superior to that attainable with a wire-micrometer. In former papers I have discussed those observations made by myself for determining the ellipticity and the form of *Saturn*, as well as those for determining the magnitude and the breadth of his bright rings; and I have thought it of importance at the present time to collect all the measures of the other planets made by myself, and to give definitively the results for all, as obtained from a series of observations made by the same person with the same instrument and in a uniform way.

"The instrument with which the observations were made was not identically the same during the whole period of time. The chief changes which occurred were in the years 1849 and 1851, when advantage was taken of some arbitrary conditions in the ratios of the focal lengths and distances of the lenses, to ensure greater flatness of the field and greater distinctness of the images near the borders. In this latter construction the power is altered by the change of the eye-glass nearest the object-glass of the telescope.

"Previously to the year 1849, the value of a revolution of the

1856MNRAS..16..139M

micrometer-screw, as determined by observations admitting of no doubt, was 17″·00. For the power used with the new eye-piece in 1849, commencing with February, the value of the revolution was assumed to be 14″·59, and, finally, for that constructed according to M. Valz's arrangement of the lenses in 1851, the value of a revolution for the power actually employed was 7″·23."

After a few additional remarks, the author proceeds to give the details of the measures which formed the groundwork of his researches. The remaining part of his paper is devoted to a discussion of those measures and the deducing of the definitive results.

Mercury.—This planet is so rarely visible in the evening, and so difficult to be measured in full daylight, that the author's materials for determining its diameter were not very abundant, although they are probably sufficient for obtaining a moderately accurate result. The definitive value of the diameter corresponding to the unit of distance is found to be 6″·89.

Venus.—In the discussion of the measures of this planet, the author takes into account the probable effects of irradiation. Considering, first, the daylight observations, he separates the measures into two classes—those made near inferior conjunction, and those near superior conjunction; and then forms two systems of equations of condition involving the constant of irradiation and the diameter of the planet as the unknown quantities. By a simple combination of the equations of condition in each set, he obtains two final equations, by the solution of which he finds the apparent true diameter of the planet corresponding to the unit of distance to be 17″·61, and the constant of irradiation to be −0″·61. A rigorous treatment of the equations by the method of least squares gives him 17″·55, and −0″·50 for the values of the same quantities. It would appear from the resulting value of the constant of irradiation, that the measured diameters are in reality too small by 0″·5, and that the contacts have been made too closely by the quantity 0″·25. This amounts, in fact, to saying, that on account of the feebleness of the light at the cusps and borders of the planet, the true cusp or border is not observed, but a point lying within it by 0″·25, a result which might have been in some degree expected. A discussion of the evening measures of the planet shows them to be totally free from the effects of irradiation,—a startling and unexpected result for a planet apparently surrounded with so large a mass of coloured diffused light.

The author next proceeds to discuss the measures for the breadth of the illuminated disk. The results for the daylight observations exhibit a most satisfactory agreement with that deduced for the measured diameters, while the evening observations show that the portion of the disk measured is also less than the true illuminated breadth, though by a smaller quantity. "Hence," says the author, "on the whole we may affirm that for evening measures of *Venus*, made with the double-image micrometer and telescope here employed, no correction is needed for the

1856MNRAS..16..139M

effects of irradiation at the limb, and that at the boundary of light and darkness the faintness of the light causes the contacts to be made at a distance 0″·36 from the point where the sun's light really touches the planet."

Mars.—The author finds the diameter of this planet at the mean distance to be 9″·84. He has also obtained $\frac{1}{62}$ for the ellipticity of the planet—a result which he considers to be a tolerable approximate to the true value.

Jupiter.—The series of observations made for the determination of the size and form of this planet is very extensive, and a synopsis of the results is given in a table, which shows the degree of agreement of the separate determinations. In this table the corrections due to the want of illumination of one or the other equatoreal portion of the disk are given, but not applied to the equatoreal diameters; but they are taken into account in the calculation of the ellipticity and of the equatoreal diameters at the standard distance. This distance has been assumed so as to be identical with M. Struve's value, in his paper on the measures of *Jupiter* and *Saturn* (*Mem. R. A. S.* vol. iii.) namely, 5·20279. The ellipticities given in the table express, as usual, the values of —

$$\frac{\text{Equatoreal diameter} - \text{Polar diameter}}{\text{Equatoreal diameter}}$$

and their general agreement is as satisfactory as can be desired, and prove abundantly that the ellipticity $\frac{1}{13 \cdot 8}$ assigned by M. Struve is too large. As an additional proof of the accuracy of his definitive value of the ellipticity, namely, 0·05934, or $\frac{1}{16 \cdot 84}$, the author has calculated by means of it and the observed value of the equatoreal diameter, on each day when oblique diameters were also observed, the values of the oblique diameters, and has compared them with the observed values. The calculation is made in the following way.

Let a and b be the equatoreal and polar semi-axes of any ellipse, and r the radius-vector (referred to the centre) inclined at an angle θ to the major axis:—

Then

$$r^2 = \frac{a^2 b^2}{a^2 \sin^2 \theta + b^2 \cos^2 \theta}$$

Let ε be the ellipticity

$$= \frac{a - b}{a} = 1 - \frac{b}{a}$$

therefore $b = (1 - \varepsilon)\, a$

and r^2 (after reduction)

$$= \frac{a^2 (1 - \varepsilon)^2}{1 - (2\varepsilon - \varepsilon^2) \cos^2 \theta}$$

whence

$$r = a(1 - \varepsilon) \left\{ 1 + \left(\varepsilon - \frac{\varepsilon^2}{2} \right) \cos^2 \theta + \frac{3}{2} \varepsilon^2 \cos^4 \theta \right\}$$

1856MNRAS..16..139M

Now if, as is the case for very nearly all the observations of oblique diameters, $\theta = 45^\circ$, then,

$$r = a(1 - \varepsilon)\left(1 + \frac{1}{2}\varepsilon + \frac{1}{8}\varepsilon^2\right)$$

or,

$$2r = 2a\left(1 - \frac{\varepsilon}{2} - \frac{3}{8}\varepsilon^2\right)$$

or oblique diameters at angle 45°

$$= \text{equatoreal diameter} \times \left(1 - \frac{\varepsilon}{2} - \frac{3}{8}\varepsilon^2\right)$$

If, then, we assume $\varepsilon = \frac{1}{16{\cdot}84}$, we have,—

$$\text{Oblique Diameter} = \text{Equatoreal Diameter} \times (1 - 0{\cdot}03102)$$

and by this formula the computed oblique diameters have been derived from the observed equatoreal diameters. These have been compared immediately with the means of the two transverse diameters corrected for phase, and the results are contained in the following table:—

Date of Observation.	Mean of Observed Transverse Diameters.	Computed Transverse Diameter.	Excess of Computed Diameter.
	″	″	″
1840, May 16	42·65	43·25	+0·60
June 10	41·70	41·30	−0·40
1841, Feb. 19	34·88	34·51	−0·37
1842, July 15	46·57	46·86	+0·29
1849, Feb. 17	42·80	43·14	+0·34
Feb. 26	42·33	42·37	+0·04
Feb. 27	42·25	42·26	+0·01
Mar. 20	40·85	40·24	−0·61
Mar. 21	40·41	40·27	−0·14
Apr. 7	39·00	38·95	−0·05

The mean of the excesses of the computed above the observed diameters is, therefore, −0″·03, which shows, in the first place, that the ellipticity has been rightly assumed, and, in the second, that the planet is strictly of a spheroidal form. Hence the author assumes definitively,—

Equatoreal diameter at distance 5·20279 = 37″·91
— — at distance unity = 197·24

$$\text{Ellipticity} = \frac{1}{16{\cdot}84}$$

After a few remarks on the general smallness of the measures of 1849, the author concludes with a synopsis of the several definitive results contained in his paper.